$$w(\xi) = w_0(\xi) + Bw_1(\xi),\ w(\xi) = -u\mu_w/(\tau_w + \tau_0)h,$$
$$w_0(\xi) = -(1-a)^{-1}[0.5(\xi^2 - 1) - a(\xi - 1)],$$
$$w_1(\xi) = -(12(1-a))^{-2}[\xi^6 - 6a\xi^5 + (6a^2 + 18a - 9)\xi^4 - (8a^3 +$$
$$+ 24a^2 - 12)\xi^3 + (12a^4 + 12a^3 - 24a + 15)\xi^2 - (24a^4 - 48a^2 +$$
$$+ 30a)\xi + 12a^4 - 4a^3 - 30a^2 + 30a - 7].$$

Here $w_0(\xi)$ is the classical solution for the isothermal flow of a viscoplastic medium obtained by Volarovich and Gutkin [7].

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HEATING IN THE DEFORMATION OF A STRUCTURED FLOWING SYSTEM

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UDC 532.1.35

The heating of a liquid by deformation may alter the flow curve (the relation between the stress and the shear rate). The relationship may become nonlinear even for a Newtonian liquid. This is related for example to the phenomenon of hydrodynamic thermal explosion [1, 2]. In the rheological processing of viscometric data, it is important to distinguish the heating effect from the effects of the internal properties of the liquid. For this purpose, either the experiment should be done under certainly isothermal conditions, which restricts the measurement range, or allowance for the heating must be made in the calculation of the characteristics. The latter is simplest to provide when there is spatial homogeneity in the temperature, which occurs for example in a constant-pressure (moment) viscometer [3].

Here we examine the behavior of a structured flowing system under conditions of heating and we distinguish the physically distinct flow states and determine the parameter ranges corresponding to the different types of rheological curve, and we also define critical conditions for structural ignition and extinction and for hydrodynamic thermal explosion in structured systems.

1. Formulation of the Problem. We consider the nonisothermal flow of a two-component liquid with mutual conversion of the components [4, 5]. The mathematical formulation includes not only the rheological and kinetic equations [4, 5] but also the heat-balance equation, which incorporates the dissipative heat production, the heat produced during the structural transformations, and the heat lost through the side walls:

$$\dot{\gamma} = \{aF_{01}\exp[\omega_1(T - T_0)] + (1 - a)F_{02}\exp[\omega_2(T - T_0)]\}\tau; \tag{1.1}$$

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$$\dot{a} = -k_{10} a \exp\left(-\frac{E_1 - p\tau - q\dot{\gamma}^2}{RT}\right) + k_{20}(1-a)\exp\left(-\frac{E_2}{RT}\right); \tag{1.2}$$

$$c\rho\dot{T} = \tau\dot{\gamma} - \alpha(S/V)(T - T_0) + Q\dot{a}, \tag{1.3}$$

where τ and $\dot{\gamma}$ are the stress and shear rate, F_{0i} are the component fluidities, a is the proportion of the component with the smaller fluidity, ω_i is a constant characterizing the activation energy of viscous flow for component i, R is the universal gas constant, T is absolute temperature, T_0 is the environmental temperature, α is the effective heat-transfer coefficient, S and V are the surface and volume of the liquid, c and ρ are specific heat and density, k_{10} and k_{20} are the rate constants for the destruction and recovery of the structure correspondingly, E_i is the activation energy for these processes, p and q are constants characterizing correspondingly the destruction rate and the orientation rotation for a structural unit, t is time, and Q is the heat of the structural transformations.

The temperature dependence of the rate constants k_i is taken in Arrhenius form:

$$k_i = k_{i0}\exp(-E_i/RT).$$

Reynold's law is used for the temperature dependence of the viscosity:

$$\eta_i = \eta_{0i}\exp[-\omega_i(T - T_0)].$$

We consider the effects of the shear deformation only on the structure disruption.

Equations (1.1)-(1.3) assume that the characteristics of the liquid do not have a spatial distribution (are homogeneous).

We introduce dimensionless variables and parameters convenient for the subsequent analysis:

$$D = \sqrt{\frac{q}{RT_0}}\,\dot{\gamma}, \quad \sigma = \sqrt{\frac{q}{RT_0}}\,F_{02}\tau, \quad \Theta = \frac{E_2(T - T_0)}{RT_0^2},$$

$$\lambda = F_{01}/F_{02}, \quad \beta = RT_0/E_2, \quad u_i = \omega_i RT_0^2/E_2, \quad \xi = (E_1 - E_2)/E_2,$$

$$\mu = \frac{p}{F_{02}\sqrt{qRT_0}}, \quad \delta = \frac{E_2}{\alpha(S/V)\,qT_0F_{02}}, \quad \varkappa = \frac{k_{10}}{k_{20}}\exp\left(-\frac{E_1 - E_2}{RT_0}\right).$$

We also introduce the characteristic times of heat loss t_0 and heat production t_1 together with the time of structural transformation t_2:

$$t_0 = c\rho/\alpha(S/V), \; t_1 = c\rho T_0 qF_{02}/E_2, \; t_2 = 1/k_{20}.$$

Then (1.1)-(1.3) take the form

$$D = [a\lambda\exp(u_1\Theta) + (1 - a)\exp(u_2\Theta)]\sigma,$$

$$t_2\dot{a} = -\varkappa a\exp\left[\frac{(\xi + 1)\Theta + \mu\sigma + D^2}{1 + \beta\Theta}\right] + (1 - a)\exp\left(\frac{\Theta}{1 + \beta\Theta}\right),$$

$$\dot{\Theta} = \sigma D/t_1 - \Theta/t_0 + Q\dot{a}E_2/(c\rho RT_0^2).$$

2. Qualitative Analysis. The qualitative analysis is naturally performed with certain simplifying assumptions. We assume that $u_2 = 1$ (this is equivalent to assuming similarity in the activation energies for viscous flow and structuring). The effects of λ and μ have been discussed previously [4, 5], and for convenience and to provide some minor simplifications we put $\lambda = \mu = 0$. We are subsequently interested in the stationary system of equations, which takes the form

$$D = (1 - a)\sigma\exp\Theta; \tag{2.1}$$

$$1 - a = a\varkappa\exp[(\xi\Theta + D^2)/(1 + \beta\Theta)]; \tag{2.2}$$

$$\Theta = \delta\sigma D. \tag{2.3}$$

Here the main attention is given to the effects on the rheological curve $\sigma(D)$ from the parameters $\varkappa$ and δ, which have the meaning of the intensity of the structural transformations and the dissipative heat production correspondingly. The limiting case $\delta = 0$ corresponds to isothermal flow and has been considered in [4, 5]. We eliminate a and Θ from (2.2) and (2.3) to get $\sigma(D)$ in the inexplicit form

$$\Phi \equiv \sigma - (k^{-1}+1)\,D\exp(-\delta\sigma D) = 0, \quad k = \varkappa\exp\left(\frac{\xi\delta\sigma D + D^2}{1+\beta\delta\sigma D}\right). \tag{2.4}$$

The condition for a turning point on the $\sigma(D)$ curve, $d\sigma/dD = 0$, and the equivalent one $\partial\Phi/\partial D = 0$ after certain transformations are put as

$$(k+1)(1-\delta\sigma D) = [D^2(2+\beta\delta\sigma D) + \xi\delta\sigma D](1+\beta\delta\sigma D)^{-2}. \tag{2.5}$$

In what follows we consider the case $0 < \xi \leqslant 1$; this range in ξ is the most interesting from the physical viewpoint. Positive values for ξ mean that in the absence of deformation the activation energy for structure destruction is greater than that for recovery. If the preexponential factors are similar, this means that an initial structure exists in liquid for $\sigma = D = 0$ (the rate of recovery is higher than the rate of destruction). On deformation, there is an effective reduction in the activation energy E_1, and the destruction rate increases, so the equilibrium shifts towards the final structure. The constraint $\xi \leqslant 1$ involves the assumption that the activation energies E_1 and E_2 differ slightly. If this is not so, the effects of the deformation are slight.

With these constraints (2.5) is always soluble and in accordance with the values of $\varkappa$ and δ can have up to three roots in the range $0 < D < 1/\delta\sigma$ (correspondingly $0 < \Theta < 1$). It will be evident from what follows that the presence of three turning points indicates that there can be a rheological curve with two maxima and one minimum.

The boundary separating the rheological curves with one and two maxima may be defined from the condition for degeneracy of the $\sigma(D)$ curve:

$$\Phi(\sigma, D) = 0, \quad \sigma'_D = 0, \quad \sigma''_{DD} = 0.$$

The latter equation and the equivalent one $\partial^2\Phi/\partial D^2 = 0$ can be put in the form

$$\begin{gathered} k\{(1-\Theta)[D^2(2+\beta\Theta) + \xi\Theta] - \Theta(1+\beta\Theta)^2\} = \\ = 4D^2 + \Theta(\xi - \beta D^2)(1-\beta\Theta)/(1+\beta\Theta) + \Theta(1+\beta\Theta)^2. \end{gathered} \tag{2.6}$$

Using (2.3), from (2.5) and (2.6) we get the explicit $D^2(\Theta)$: relationship:

$$\begin{gathered} D^2 = -A + \sqrt{A^2 - B}, \quad A = [\xi\Theta - (1-\Theta)(1+\beta\Theta)/2 - \\ - \Theta(1+\beta\Theta)^2/2(1-\Theta) - 2(1+\beta\Theta)^2/(2+\beta\Theta) + \beta\Theta(1-\beta^2\Theta^2)/ \\ /2(2+\beta\Theta)]/(2+\beta\Theta), \\ B = \xi\,\frac{\xi\Theta^2(1+\beta\Theta) - \Theta(1-\Theta)(1+\beta\Theta)^2 + \Theta^2(1+\beta\Theta)^2(1-\Theta) + (1-\beta^2\Theta^2)\Theta}{(2+\beta\Theta)^2}, \end{gathered} \tag{2.7}$$

which enables us to derive the boundary in parametric form (parameter Θ):

$$\varkappa = k\exp\left[-\frac{\xi\Theta + D^2(\Theta)}{1+\beta\Theta}\right]; \tag{2.8}$$

$$\delta = (k^{-1}+1)D^{-2}(\Theta)\ \Theta\exp\Theta. \tag{2.9}$$

In the relevant range in $\xi\ (0 < \xi \leqslant 1)$, the expression in the radical in (2.7) is positive. In fact, for $\beta = 0$ the range of values where $A^2 - B \geqslant 0$ is described by the following:

$$\xi \leqslant (\Theta^2 - 3\Theta + 3)^2/[4\Theta(1-\Theta)^2] = \xi^*(\Theta).$$

The function ξ^* in the region $\Theta < 1$ has the minimum $\xi_0^* = 6.125$; if one incorporates $\beta \neq 0$ numerical calculations show that ξ_0^* increases. It can also be shown that solutions of the form $A - \sqrt{A^2 - B}$ are negative in this range in ξ.

3. Rheological Curve under Self-Heating Conditions. From (2.4) we have the following limiting relations:

$$\lim_{D\to 0}\sigma(D) = 0, \quad \lim_{D\to\infty}\sigma(D) = 0.$$

The second relation means that there is a difference from the isothermal case, where $\sigma(D) \to \infty$ for $D \to \infty$, in that in a nonisothermal flow there is a reduction in the deformation resistance as D increases in the range of sufficiently high shear rates [6].

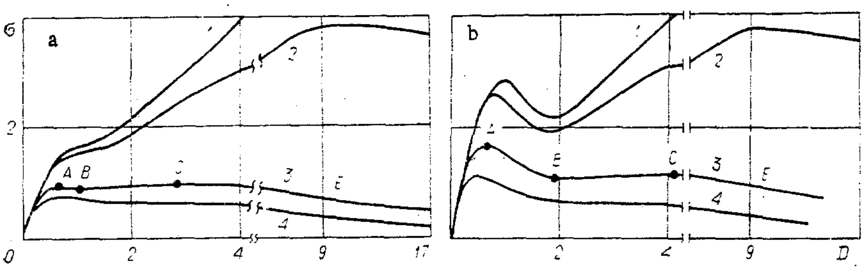

Fig. 1

This reduction may also be due to the structural transformations in a certain range in D [4, 5]. There can thus be separate or combined effects from the structural and thermal factors.

Parts a and b of Fig. 1 show the changes occurring in the rheological curve as δ increases (the increase is with the number of the curve, $\delta = 0.025$ and 0.75 for curves 2 and 4 correspondingly). Curves 1 in parts a and b of Fig. 1 correspond to isothermal flow ($\delta = 0$) but different values of $\varkappa$ ($\varkappa = 0.6 > \varkappa_* (\lambda)$ in Fig. 1a, and $\varkappa = 0.2 < \varkappa_* (\lambda)$ in Fig. 1b, and see [4, 5] on $\varkappa_* (\lambda)$).

In the general case, the $\sigma(D)$ curve has two maxima (curve 3 in Fig. 1a and b, $\delta = 0.37$ and 0.25 correspondingly). The first of them (at the smaller D) is due to the structural transformations, while the second is due to the nonisothermal flow [4-6]. The minimum on the curve is due to saturation in the structural transformations. This case occurs if the temperature-dependent nonlinearity in the viscosity makes itself felt after the structural saturation. If the temperature-dependent viscosity nonlinearity appears earlier during self-heating, the rheological curve can have only one turning point (curves 4 in Fig. 1a and b).

Different forms for the rheological curves can be obtained by changing the thermal parameters δ, the boundary of (2.8) and (2.9) separating the $\varkappa$, δ parameter plane into regions corresponding to each type of curve. Figure 2a shows a calculation on this boundary. In region II, one gets a $\sigma(D)$ curve with two maxima, while in region I there is one. When δ changes, in accordance with the value of $\varkappa$ one can get II-I transitions ($\varkappa < \varkappa_*(\lambda)$) and I-II-I ones ($\varkappa > \varkappa_*(\lambda)$) . In the latter case, on deformation under nonisothermal conditions one can get a structural maximum on the rheological curve, although there is none such in the isothermal case (curves 1-3 of Fig. 1b).

The individual parts of the rheological curve correspond to physically different modes of flow in the structured liquid. As the boundary between them we take the coordinates of the turning points on $\sigma(D)$ (points A, B, and C on curves 3 of Fig. 1a and b). Then branch OA corresponds to isothermal flow, AB to the extensive structural process, BC to flow with a maximally destroyed structure, and CE to nonisothermal flow. Note that in the state D = constant, all these flow modes are realized in sequence as D increases.

One can solve (2.3)-(2.5) together to determine the parameter ranges corresponding to the different flow modes. With $\varkappa$ fixed ($\varkappa = 0{,}2$ and $\varkappa = 0{,}6$), in accordance with the modes D = constant and σ = constant, these regions may be constructed in the D–δ and σ–δ parameter planes. The extreme branches of the $D(\delta)$ curve in Fig. 2b and c, correspond to maxima, while the intermediate one corresponds to a minimum on $\sigma(D)$. The $D(\delta)$ curve and the straight lines $\delta = \delta_1$ and $\delta = \delta_2$ (points where the turning points fuse) separate regions where the structural and thermal factors influence the curve separately. These regions correspond to distinct flow states: region 1 the isothermal state, 2 the structured state, 3 the state with maximally destroyed structure, and 4 the nonisothermal one. In the regions $\delta < \delta_1$ and $\delta > \delta_2$ (Fig. 2b and c), one cannot distinguish characteristic states, which indicates that the structural and thermal factors influence the flow curve jointly.

An important point is that the $D(\delta)$ diagram for the state D = constant enables one to indicate the measurement range approximately corresponding to isothermal conditions: region 1. The boundary of this region shows that this range increases as δ decreases, which can be obtained by reducing the gap between the cylinders in a rotational viscometer, and also by increasing the environmental temperature.

4. Critical Phenomena. In the state σ = constant under nonisothermal conditions, one can also get critical phenomena such as hydrodynamic thermal explosion [1, 2] (the stationary

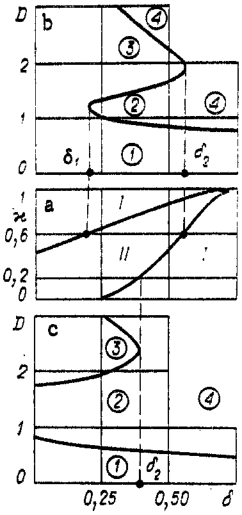

Fig. 2

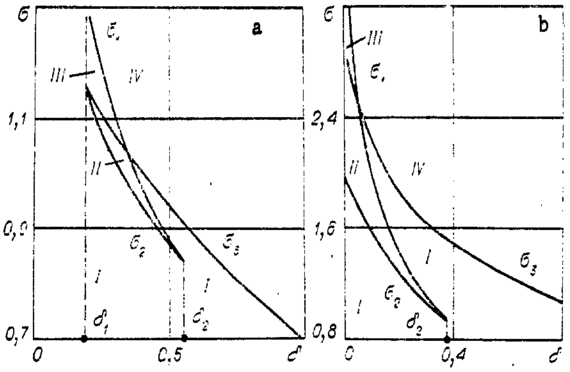

Fig. 3

state vanishes) or the hydrodynamic analogs of ignition and extinction [6, 7] (step transitions from one stationary state to another with small changes in the parameters). Here usually the critical conditions for the forward and reverse transitions do not coincide and there is hysteresis. Such transitions are possible even in the isothermal case for a structured liquid [4, 5] and are due to the structural transformations. This critical phenomenon with hysteresis may be called structural ignition and extinction from the analogy.

Nonisothermal flow in a structured liquid causes the self-heating to reduce the viscosity and intensify the structural transformations. There are two mechanisms whereby the heating influences this process. First, there is the kinetic one, where the structural process is accelerated because the rate constants are dependent on temperature. Secondly, there the hydrodynamic one, in which the changes in the hydrodynamic characteristics (σ and D) because of the temperature dependence of viscosity also accelerate the structural process. These mechanisms result for example in a region where the stationary states are not unique, which is characteristic of ignition-extinction phenomena (region II in Fig. 3a and b) for $\delta > \delta_1$, where the stationary state is unique in the isothermal case ($\delta = 0$).

The $\sigma_1(\delta)$ and $\sigma_3(\delta)$ curves in parts a and b of Fig. 3 represent the loci correspondingly of the thermal and structural maxima, while $\sigma_2(\delta)$ represents the minimum. Together with the straight lines $\delta = \delta_1$ and $\delta = \delta_2$, these curves distinguish regions corresponding to different flow modes: region I of low-temperature flow, II nonunique stationary states, III flow with maximally disrupted structure, and IV the absence of stationary modes.

There is always a unique stationary state here when D = constant, whereas if σ = constant we see that there may be nonuniqueness or the absence of stationary states. The $\sigma_1(\delta)$, $\sigma_2(\delta)$, and $\sigma_3(\delta)$ curves (Fig. 3) define the critical conditions for changes between modes. In particular, the critical condition for loss of the stationary state (explosion) is defined by the curve

$$\sigma_*(\delta) = \max\{\sigma_1(\delta),\ \sigma_3(\delta)\}$$

(boundary of region IV).

One can speak of hydrodynamic thermal explosion HTE only in the region $\sigma_1(\delta) > \sigma_3(\delta)$, where the structural transformations have time to occur long before the onset of effects from the nonisothermal flow, and one can calculate the HTE critical conditions with considerable accuracy (error less than 0.1%) by means of the expression $\sigma_*(\delta) = (e\delta)^{-1/2}$, which has been derived for a Newtonian liquid [6]. If the inequality is reversed, the critical transition into region IV is due to the thermal and structural factors. Then from (2.3), (2.8), and (2.9), which define the critical conditions for HTE, we get the asymptote $\sigma_*(\delta)$ for large δ:

$$\sigma_*(\delta) = \sqrt{s_0/(\delta + s_1)},\quad s_0 = \Theta_0(k_0^{-1} + 1)\exp(-\Theta_0),\quad k_0 = \varkappa \exp\Theta_0 \tag{4.1}$$

The temperature rise preceding explosion can be calculated from

$$\Theta_*(\delta) = \Theta_0 - \Theta_1/(\delta + s_1),$$

$$\Theta_1 = \Theta_0^2 s_0^{-2}(k_0+1)^{-1}\left[\frac{\Theta_0 k_0}{k_0+1} - 2\right]\left[\frac{1}{\Theta_0} - \frac{\Theta_0 k_0 \xi}{(k_0+1)^2}\right]^{-1}, \tag{4.2}$$

$$s_1 = s_0\Theta_0\left(\frac{1}{\Theta_0} - 1 - \frac{\xi}{k_0+1}\right) - \frac{\Theta_0/s_0^2}{k_0+1}.$$

The value of Θ_0 is found by solving the transcendental equation

$$\Theta_0 = [1 + \xi/(k_0 + 1)]^{-1}.$$

In practice, (4.1) and (4.2) describe $\sigma_*(\delta)$ and $\Theta_*(\delta)$ satisfactorily for $\delta \geqslant 0.2$ (error less than 0.5%).

Figures 1-3 have been given for the case $\beta = 0$ and $\xi = 1$, but incorporating $\beta \neq 0$ does not result in any substantial quantitative changes in the curves (the maximum relative deviation does not exceed β).

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MOTION OF A CLOUD OF HEATED PARTICLES ABOVE A HORIZONTAL SURFACE IN AN EXTERNAL FORCE FIELD

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The motion of a system (cloud) of particles in an external force (gravity) has been studied experimentally and theoretically in the isothermal case where the temperatures of the particles and carrier medium are the same; a review and bibliography is given in [1, 2]; see also [3, 4]. One of the basic features of these studies was the identification of two different types of motion of the cloud depending on the degree of hydrodynamical or gas-dynamical interaction between particles via the carrier phase. In the "filtration" regime this interaction is small and each particle in the cloud moves independently. In the "entrainment" regime, because of the friction between the phases, large-scale motion (of the order of the size of the cloud) of the dispersed medium with a rising flow on the periphery arises and the precipitating cloud is transformed into a vortex ring with continuously increasing diameter.

Many phenomena in nature and in technological processes are accompanied by the formation of aerosol clouds in which the temperature of the particles is higher than that of the ambient medium (emission from a smokestack, combustion products in fires, emission of aerosols in the eruption of volcanoes). The initial temperature differential leads to new features in the evolution of the cloud of particles. As a result of interphase heat exchange the gas in the cloud is heated and expands, carrying along particles with it. As a result, for a sufficiently high concentration of particles, the cloud size increases in the initial

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